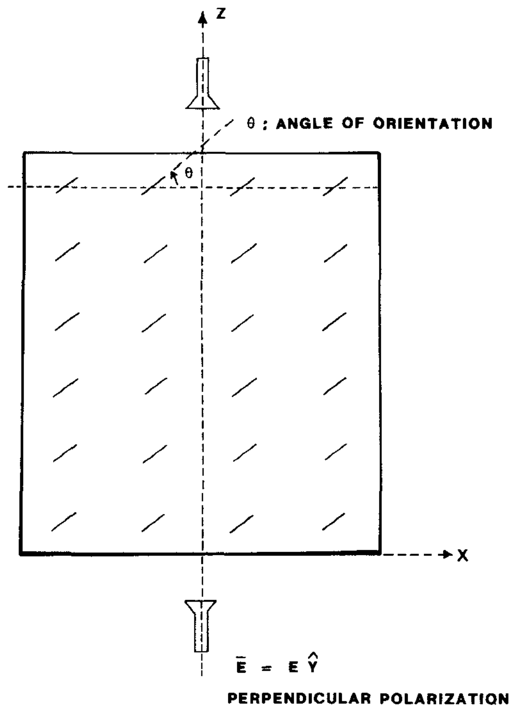

PLAN VIEW

Fig. 6. Strip simulation of an anisotropic plasma. Adjustment of the orientation of the strips can simulate the required anisotropy.

in fusion machines [17], is presently under investigation by the first author. It appears the strips are more suitable than the rods for simulating the anisotropic plasma. The orientation of the strips (Fig. 6) with reference to the direction of propagation can simulate the required anisotropy.

VII. Summary

In this paper, a technique is developed for the simulation of the lossless and low-loss plasma, using the two-dimensional strip medium. The plasma parameters (plasma frequency ω_p and collision frequency ν) are related to the parameters of the strip medium (width of the strip (w), thickness of the strip (t), separation between two strips (a), and spacing between two successive planes in the direction of propagation (b)).

The necessary conditions for the plasma simulation are: 1) the electric vector of the incident wave should be parallel to the strips; 2) the spacing between two successive planes of strips in the direction of propagation should be less than the separation between strips in the transverse plane to avoid the reactive coupling between adjacent planes of elements; 3) the width of the strips should be less than the separation between strips; and 4) the separation between strips should be less than half of the free-space wave length.

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Microwave Radiation from a Magnetic Dipole in an Azimuthally Magnetized Ferrite Cylinder

R. S. MUELLER

Abstract—**The electromagnetic radiation pattern from a ferrite coated dipole antenna is a torus with a point center. By deriving an expression for the far-field electric field and defining a form factor $F(\theta)$, the dilations and contractions of the radiation pattern were evaluated and demonstrated graphically.**

I. Introduction

Almost all the work on ferrite antennas has been experimental. Attempts to explain the radiation patterns of ferrite radiators have been based on an analysis by Kiely [1] who considered dielectric rods in the hybrid HE_{11} mode. The present paper is concerned with radiation from a magnetic dipole in an azimuthally magnetized ferrite cylinder. Similar problems are found in the literature on antennas immersed in plasmas or with ferrite coatings. The number of these articles [2]–[10] increased when plasma effects began to interrupt communications with space vehicles reentering the Earth's atmosphere.

An oscillating magnetic dipole in a column of azimuthally magnetized ferrite may act as a waveguide and as a radially radiating antenna. A magnetic dipole along the z-axis consists of a loop antenna in the x-y plane with a sinusoidal circulatory current. The loop antenna is a less effective radiator than an electric dipole antenna of the same size and driving current. However, at low frequencies the electric dipole requires a higher

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driving voltage to produce a significant current distribution along its length. The choice of a magnetic dipole rather than an electric one was made because the magnetic dipole excites the azimuthally-symmetric, magnetically dependent TE modes [11] along the cylinder. An electric dipole supports the magnetically independent TM modes.

Azimuthal magnetization could be achieved with a small current carrying center conductor. Ampere's law predicts that the magnetic field in the ferrite would be $H_t = I/2\pi\rho$ where I is the current in the conductor and ρ is the radial cylindrical coordinate ($a > \rho > b$). The outer and inner radii of the ferrite tube are a and b. Although the magnetic field varies with distance from the axis, if $a-b$, the thickness of the ferrite tube, is small compared with $1/2(a+b)$, the mean radius, then the magnetic field will be approximately uniform across the ferrite with the value $H_t = I/\pi(a+b)$. The behavior of a dipole in an azimuthally magnetized ferrite cylinder described here assumes that there is no radial dependence of the applied field.

The boundary value problem was solved using exponential transforms of the field equations. To find the radiation pattern the inverse transform was needed. However, the inverse transform expression involves an integral with a very complicated transcendental integrand. The saddle-point method of evaluating contour integrals was applied to find the far-field radiation pattern.

II. The Azimuthally Magnetized Cylinder

The radiator shown in Fig. 1 consists of a uniformly magnetized ferrite cylinder of radius a coaxial with the z-axis. The magnetic dipole source or feed is also oriented along the z-axis and is located at the origin of the set of cylindrical coordinates ρ, ϕ, and z.

The magnetic dipole vector may be expressed as a product [5] of delta functions:

$$\mathbf{M} = \hat{z}M_z \qquad M_z = \tfrac{1}{2}M_e\delta(\rho)\delta(z)/\pi\rho.$$

Without loss of generality, the electromagnetic fields were assumed to vary as $\exp(-j\omega t)$ where ω is the angular frequency and t is time. The electromagnetic field equations from the small signal theory of microwave ferrites are written:

$$\nabla \mathbf{X} H = -j\omega\epsilon_0\epsilon_f \mathbf{E}$$

$$\nabla \mathbf{X} E = j\omega\mu_0 \boldsymbol{\mu} \mathbf{H} + \mathbf{M}$$

where ϵ_f is the relative permittivity and j is the positive square root of minus one. The microwave excitation frequency is given by f, the precessional frequency f_0 by βH_t, and the magnetization frequency f_m by βM_s where β is the gyromagnetic ratio and M_s is the saturation magnetization.

The relative permeability tensor $\boldsymbol{\mu}$ of an azimuthally magnetized ferrite is given by

$$\boldsymbol{\mu} = \begin{bmatrix} \mu & 0 & j\kappa \\ 0 & 1 & 0 \\ -j\kappa & 0 & \mu \end{bmatrix}$$

where

$$\mu = 1 + \frac{f_0 f_m}{f_0^2 - f^2} \qquad \kappa = \frac{f f_m}{f_0^2 - f^2}.$$

Since the magnetic dipole is azimuthally symmetric, all the electromagnetic fields are independent of ϕ. Therefore, it may be assumed that the electromagnetic field solutions may be expressed in the form

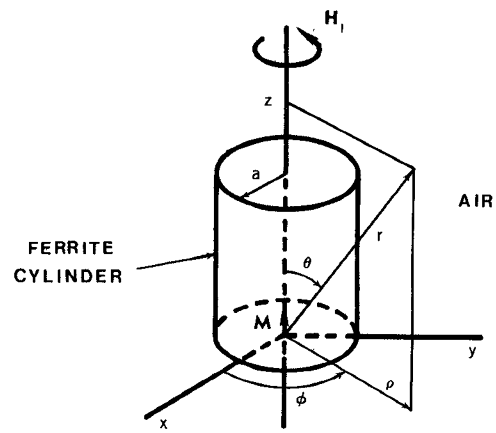

Fig. 1. A magnetic dipole in a cylinder of azimuthally magnetized ferrite.

$$f(\rho, z) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} F(\rho, \zeta)e^{j\zeta z}\,d\zeta$$

$$F(\rho, \zeta) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} f(\rho, z)e^{-j\zeta z}\,dz.$$

Solutions to the wave equation for the fields in the ferrite region ($\rho < a$) can be found [11] in the literature:

$$E_\phi(\rho, \zeta) = \tfrac{1}{4}j(1+\alpha^2)m_e k\left[M_1^{(1)}(\alpha; k\rho) + L(\zeta)B_1(\alpha; k\rho)\right]$$

where

$$B_1(\alpha; x) = a_0 x + a_1 x^2 - \sum_{n=0}^{\infty} \frac{a_n + \alpha a_{n+1}}{(n+2)(n+4)} x^{n+3}$$

$$a_0 = \frac{1}{2} \quad a_1 = -\frac{\alpha}{6}$$

$$H_1(\alpha; x) = \frac{2}{\pi}\left\{\left[\ln\frac{x}{2} + \gamma\right]B_1(\alpha; x) + \sum_{n=0}^{\infty} d_n x^{n-1}\right\}$$

$$d_0 = -\frac{1}{\alpha^2+1} \quad d_1 = -\frac{\alpha}{\alpha^2+1} \quad d_2 = -\frac{1}{4}$$

$$d_{n+2} = -\frac{2(n+1)a_n + d_n + \alpha d_{n+1}}{n(n+2)}$$

$$\gamma = \text{Euler's constant}.$$

The function $M_1^{(1)}(\alpha; x)$ is the complex sum of $B_1(\alpha; x)$ and $H_1(\alpha; x)$:

$$M_1^{(1)}(\alpha; x) = B_1(\alpha; x) + jH_1(\alpha; x).$$

The undetermined constant $L(\zeta)$ can be found from the boundary conditions and m_e is the normalized amplitude of the magnetic dipole $(2\pi)^{-1/2}M_e$. The field component $H_z(\rho, \zeta)$ is given by

$$H_z(\rho, \zeta) = \tfrac{1}{4}(1+\alpha^2)m_e k\left\{\zeta\kappa\left[M_1^{(1)}(\alpha; k\rho) + L(\zeta)B_1(\alpha; k\rho)\right] + k\mu\left[M_0^{(1)}(\alpha; k\rho) + L(\zeta)B_0(\alpha; k\rho)\right]\right\}/\gamma_0^2$$

where $\alpha = -\zeta\kappa/k\mu$, $k^2 = \omega^2\epsilon\mu_0(\mu^2-\kappa^2)/\mu - \zeta^2$, and $\gamma_0^2 = \omega\mu_0(\mu^2-\kappa^2)$. The radial propagation constant in the ferrite is k. The zero-order functions are defined as follows:

$$\frac{1}{\rho}\frac{d}{d\rho}\left[\rho\Lambda_1(\alpha; k\rho)\right] = k\Lambda_0(\alpha; k\rho).$$

The quantity Λ denotes B, H, or $M^{(1)}$ or any linear combination of these functions.

The electromagnetic fields in the adjoining air region are

$$\mathrm{E}_{\phi}(\rho,\zeta)=\tfrac{1}{4}j(1+\alpha^2)\mathrm{m}_e D(\zeta)H_1^{(1)}(\xi_0\rho)$$
$$\mathrm{H}_z(\rho,\zeta)=(1+\alpha^2)\mathrm{m}_e\xi_0 D(\zeta)H_0^{(1)}(\xi_0\rho)/4\omega\mu_0 \qquad (1)$$

where $\xi_0^2=k_0^2-\zeta^2$. The quantity $D(\zeta)$ may be determined by equating the tangential components of the field vectors in the ferrite to those in the air at $\rho=a$:

$$D'(\zeta)=\mu k^3\left[B_0(\alpha;ka)M_1^{(1)}(\alpha;ka)-B_1(\alpha;ka)M_0^{(1)}(\alpha;ka)\right]/\gamma_0^2$$
$$\Delta(\zeta)=-k\xi_0B_1(\alpha;ka)H_0^{(1)}(\xi_0a)/\omega\mu_0+k\left[\kappa\zeta B_1(\alpha;ka)+\mu kB_0(\alpha;ka)\right]H_1^{(1)}(\xi_0a)/\gamma_0^2$$

where $D'=D\Delta$. To find $\mathrm{E}_{\phi}(\rho,z)$, take the inverse transform of (1):

$$\mathrm{E}_{\phi}(\rho,z)=j(32\pi)^{-1/2}(1+\alpha^2)\mathrm{m}_e\cdot\int_{-\infty}^{\infty}D'(\zeta)\Delta^{-1}H_1^{(1)}(\xi_0\rho)e^{j\zeta z}\,d\zeta. \qquad (2)$$

The poles of the integral expression are given by the roots of $\Delta(\zeta)=0$. The real ζ roots satisfy the following equation:

$$\xi_0B_1(\alpha;ka)H_0^{(1)}(\xi_0a)/\omega\mu_0-\left[\kappa\zeta B_1(\alpha;ka)+\mu kB_0(\alpha;ka)\right]H_1^{(1)}(\xi_0a)/\gamma_0^2=0.$$

The Hankel functions are complex numbers for real values of ξ_0a. Therefore, the left-hand side is complex in general, even if k is real or imaginary. The real ζ roots of the equation can exist only when the equation has all real terms. This will be true only when ξ_0a is purely imaginary since $H_0^{(1)}(jx)$ is imaginary and $H_1^{(1)}(jx)$ is real. Physically when ξ_0^2 is positive the magnetic dipole will develop a toroidal field pattern in the air. When ξ_0^2 is negative the ferrite tube will act like a waveguide for longitudinal propagation and the fields in the air will decay rapidly away from the boundary surface.

Without the dipole source [12] the boundary conditions at the air-ferrite interface cannot be satisfied for $k^2<0$. There are natural modes or propagation along the cylinder when $k^2>0$ and evanescent behavior for $k^2<0$. By making the substitution $-\xi_1^2$ for ξ_0^2, the secular equation that describes the natural waves propagating along the z-axis is obtained:

$$\xi_1B_1(\alpha;ka)K_0(\xi_1a)/\omega\mu_0+\left[\kappa\zeta B_1(\alpha;ka)+\mu kB_0(\alpha;ka)\right]K_1(\xi_1a)/\gamma_0^2=0.$$

Equation (2) may be used to find the far-field radiation pattern when $\xi_0\rho\gg1$ and $\xi_0^2>0$. To evaluate this integral, first make the following coordinate transformation from cylindrical to spherical polar coordinates and replace the Hankel function with its asymptotic expansion:

$$\zeta=k_0\cos\psi \quad \xi_0=k_0\sin\psi \quad z=r\cos\theta \quad\text{and}\quad \rho=r\sin\theta$$
$$H_1^{(1)}(\xi_0\rho)\simeq(2/\pi\xi_0\rho)^{1/2}e^{j[\xi_0\rho-(3/4)\pi]}$$

where r is the polar radius and θ is the polar angle. From these

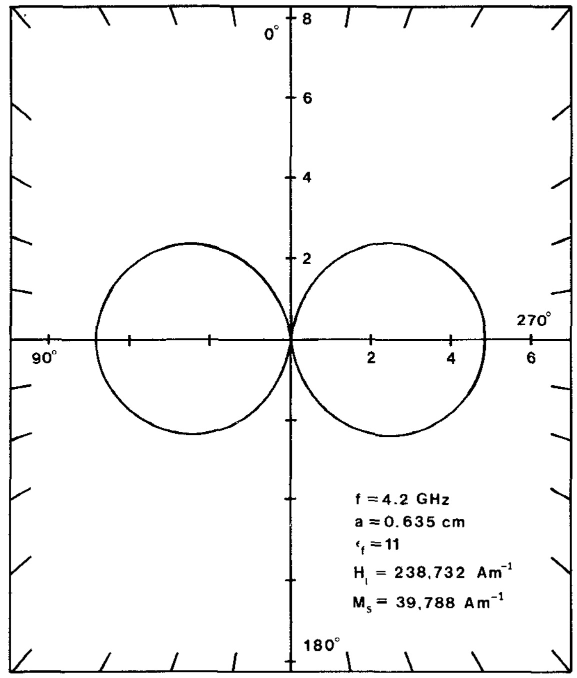

Fig. 2. Polar plot of form factor $F(\theta)$.

relations one finds

$$\mathrm{E}_{\phi}(r,\theta)=-j(1+\alpha^2)\mathrm{M}_e\left(k_0/32\pi^3r\sin\theta\right)^{1/2}\cdot\int_{\pi-j\infty}^{j\infty}D'\Delta^{-1}e^{j[k_0r\cos(\theta-\psi)-(3/4)\pi]}\sin^{1/2}\psi\,d\psi.$$

The asymptotic expansion of the integral according to the saddle-point method [13] of evaluating contour integrals yields

$$\mathrm{E}_{\phi}(r,\theta)=-j(1+\alpha^2)\mathrm{M}_eD'(\zeta_0)e^{jk_0r}/4\pi r\Delta(\zeta_0)$$

where $\zeta_0=k_0\cos\theta$. The magnitude of the field is

$$|\mathrm{E}_{\phi}(r,\theta)|=(1+\alpha^2)k_0\mathrm{M}_eF(\theta)/4\pi r. \qquad (3)$$

The shape of the radiation pattern is given by the form factor $F(\theta)$:

$$F(\theta)=|D'(\zeta_0)/k_0\Delta(\zeta_0)| \qquad (4)$$

$$|D'(\zeta_0)|=|k^3\mu\left[B_1(\alpha;ka)H_0(\alpha;ka)-B_0(\alpha;ka)H_1(\alpha;ka)\right]/\gamma_0^2|$$

$$|\Delta(\zeta_0)|=\Big[\big\{kk_0\sin\theta B_1(\alpha;ka)J_0(\xi_0a)/\omega\mu_0-k\left[\kappa\zeta_0B_1(\alpha;ka)+\mu kB_0(\alpha;ka)\right]J_1(\xi_0a)/\gamma_0^2\big\}^2+\big\{kk_0\sin\theta B_1(\alpha;ka)N_0(\xi_0a)/\omega\mu_0-k\left[\kappa\zeta_0B_1(\alpha;ka)+\mu kB_0(\alpha;ka)\right]\cdot N_1(\xi_0a)/\gamma_0^2\big\}^2\Big]^{1/2}.$$

The three-dimensional radiation pattern is a torus with a point center. Its cross section has the lemniscate shape shown in Fig. 2.

The form factor $F(90°)$ was used to study the dilations and contractions of the pattern with respect to frequency. It can be arranged in the following convenient form for amplitude analysis:

$$F(90^\circ)=\frac{(k/k_0)|J_0(ka)N_1(ka)-J_1(ka)N_0(ka)|}{\left\{\left[J_0(ka)J_1(\xi_0a)-(k/\epsilon_fk_0)J_1(ka)J_0(\xi_0a)\right]^2+\left[J_0(ka)N_1(\xi_0a)-(k/\epsilon_fk_0)J_1(ka)N_0(\xi_0a)\right]^2\right\}^{1/2}}.$$

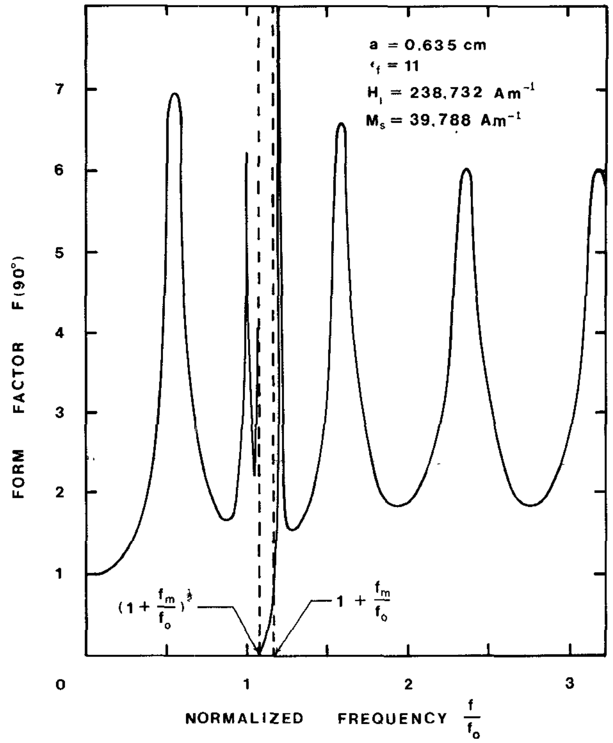

Fig. 3. Antenna pattern form factor $F(90°)$ versus normalized frequency.

For evaluation, a ferrite rod of radius 0.25 in (0.635 cm), saturation magnetization of 39 788 Am^{-1}, azimuthal magnetic field of 238 732 Am^{-1}, and relative permittivity of 11 was used.

Fig. 3 shows a series of resonant peaks and troughs. This behavior is characteristic of the geometry more than the magnetic properties of the ferrite. A peak occurs approximately at $f/f_0 = (3/4+n)c/2af_0\epsilon_f^{1/2}$ where n is an integer. The salient properties that affect the form factor are the dielectric constant and radius of the ferrite cylinder. If a wavelength λ is defined to be $c/f\epsilon_f^{1/2}$ then the radius can be expressed as $a=(3/4+n)\lambda/2$. One can interpret the peak/trough behavior as body resonances which occur when the ferrite cylinder has a radius equal to $3\lambda/8$ plus an integral number of half wavelengths.

In Fig. 3, the peak separation is $\Delta f = 0.8479 f_0$. Near the region between $1.0801 f_0$ and $1.1667 f_0$ the characteristic oscillates rapidly in magnitude, dropping to zero at $1.0801 f_0$. This is due to the rapid sign and magnitude changes of the factor $(k/k_0)J_1(ka)$ in the denominator at $F(90°)$.

III. Radiation from a Two Element Array

Antenna arrays make use of wave interference phenomena that occurs between the radiation from different elements [14] of the array. Consider a two element array of magnetic dipoles in two azimuthally magnetized ferrite cylinders labeled 0 and 1 and nondirectional in the plane under consideration. The vector sum of the fields at an arbitrary point in space, sufficiently remote from the antenna system will be $\mathrm{E} = \mathrm{E}_\phi(1+qe^{j\Omega})$ where E_ϕ is the field intensity due to antenna 0 alone, q is the ratio of the magnitudes of the intensities, and Ω is the phase difference between the two radiators. The magnitudes can be controlled by tuning the static magnetic fields in the ferrite cylinders. The magnitude of the total field intensity is given by

$$\mathrm{E}_T = \mathrm{E}_\phi\left[(1+q\cos\Omega)^2 + q^2\sin^2\Omega\right]^{1/2}. \tag{5}$$

From (3), it can be seen that the magnitude of E_ϕ resulting from antenna 0 is directly proportional to a form factor F_0 and the magnitude of E_ϕ from antenna 1 is proportional to F_1. Therefore, q is the ratio F_1/F_0. When the intensities are equal then E_T is equal to $2\mathrm{E}_\phi\cos\frac{1}{2}\Omega$.

IV. Conclusions

The present paper has analytically explained the behavior of one of the elements of an antenna array consisting of azimuthally magnetized ferrite rods excited by magnetic dipoles. One of the results of the analysis is a form factor that defines the radiation pattern in terms of the electromagnetic properties of the ferrite rod. The basic pattern is toroidal but the properties of the ferrite rod make it adjustable.

The form factor $F(\theta)$ is resonant in nature exhibiting a large resonant peak at low frequencies and a regular series of peaks and troughs at higher frequencies. The radiation from some elements may be amplified and some attenuated by tuning each element of the array separately with its individual azimuthal magnetic field. In this way, elements may even be turned off completely when only a selected number of them are required to radiate.

Further study of magnetic dipole ferrite rod antennas may use the general expression of the form factor to synthesize arbitrary radiation patterns. Equation (5) gives the combination of the field intensities from two nondirectional ferrite rod radiators.

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